XIV. *On a Method of examining the Divisions of astronomical Instruments. By the Rev.* William Lax, *A. M. F. R. S.* Lowndes's *Professor of Astronomy in the University of Cambridge. In a Letter to the Rev. Dr.* Maskelyne, *F. R. S. Astronomer Royal.*

Read June 1, 1809.

DEAR SIR,

St. Ibbs, Aug. 27, 1808.

I AM persuaded that you must feel, in common with myself, how unpleasant it is to make use of an instrument in astronomical observations requiring extreme accuracy, whose exactness you have no adequate means of ascertaining, but are obliged to depend for it in a great measure upon the abilities and integrity of the artist. It is in vain that we observe with so much nicety, and read off with so much precision, if we are still uncertain whether there may not be an error in the instrument itself of much greater magnitude, than those which we are endeavouring to prevent; and that our best instruments must be liable to such errors, no person can possibly doubt, who has paid due attention to the sources from whence they may arise. I have estimated, as accurately as I could, the amount to which they may accumulate in BIRD's method of dividing by continual bisections, and have satisfied myself that they are much more considerable than is generally apprehended; but as I cannot obtain such precise information as I could wish, respecting the exactness with which a bisection can be performed, or a length taken from the scale of equal

parts and laid upon the instrument, I will not trouble you with the deduction which I have made. It is understood indeed, that BIRD's method is now generally laid aside, and that each artist employs one, which he considers in many respects as peculiar to himself; but I presumed that there would still be such a connection betwixt BIRD's method and those which have been substituted in its stead, as to render them in some degree liable to the same errors to which it was subject, and the reports which I have uniformly received from persons, who have had an opportunity of examining some of the modern instruments, have fully convinced me that my opinion was just. But whatever may be the nature of the methods which are now in use, or whatever their advantages over BIRD's, I never could persuade myself that it would be safe to trust to an instrument, without a previous examination. To discover the means of accomplishing this object, is what I have for some time been anxious to effect, and though I fear my endeavours have not been very successful, I will nevertheless take the liberty of presenting you with the result.

You are aware, I believe, that I use a circular instrument for observing both in altitude and azimuth, which was made for me by Mr. CARY in the Strand; that the radius of both the altitude and the azimuth circle is one foot, and that each is divided into parts containing ten minutes. The construction of this instrument does not differ materially from that of other similar instruments, with which you are well acquainted, and I shall not therefore waste your time by giving you a particular description of it. For the purpose of examining the divisions upon the two circles, I procured an apparatus to be prepared by Mr. CARY, which will be very easily explained.

To the face of the rim which surrounds the azimuth circle, and with its left end close to the stand which supports the micrometer on the east side, an arc of brass, concentric with the circle itself, and a little more than 90° in length, an inch in breadth, and one eighth of an inch in thickness, is firmly fixed by screws, so as to have the plane parallel to the plane of the circle, and a small portion of its lower surface resting upon the extreme part of the rim. The screws pass through a brass arc, which is fastened to this at right angles, and lies with its broad side against the face of the rim. Upon the first mentioned arc, a strong upright piece of brass, about six inches in length, is made to slide, the lower part of it embracing the arc as a groove and having a clamping screw underneath, for the purpose of fixing it firmly to the arc at any point required. To the top of the upright piece of brass is attached a microscope, with a moveable wire in its focus, pointing down to the division upon the circle, not directly, however, but with an inclination to the left of about 30°. This inclination is given to it, in order to make it point to the same division upon the circle, which is immediately under the micrometer itself, when it has been moved up as near to the micrometer, as it is capable of approaching. The microscope has attached to it a small graduated circle of brass, and an index, by which the seconds, and parts of a second, moved over by the wire are determined.

To the vertical circle there is likewise an arc applied, of the same length and breadth as the former, but somewhat thicker, and of a radius exceeding that of the circle by about two inches. This greater thickness is given to it, on account of its being supported in a manner which renders additional

strength necessary. It is fixed with its broad convex side downwards upon two brass pillars, screwed fast to the plane of the azimuth circle, and standing in a line parallel to the plane of the vertical circle at the distance of about four inches from it, and on the right side of the pillars which support the micrometers belonging to this circle. The pillar, to which the left end of the arc is fastened, is placed close to the lower micrometer of the vertical circle, and the other contiguous to the elevated rim, in which the divisions of the azimuth circle are cut. The right end of the arc reaches beyond this pillar about ten inches. The pillars are of such a height, and so proportioned to each other, that whilst the left end of the arc, which lies horizontally, is raised to within about two inches of the height at which the lowest point of the vertical circle is placed, the whole arc runs parallel to the circle through an extent of something more than 90°. Upon the arc a microscope, with a moveable wire in the focus, is made to slide as in the former case, and to point to the divisions upon the vertical circle, not directly, but with an inclination of about 30° to the left, in order that the same division (which is the lowest upon the circle) may be seen through it, and through the lower micrometer at the same time.

I will now proceed to shew you, in what manner the examination of the divisions upon either circle may be performed. The process is precisely the same in both cases, and will of course be described in the same words.

The first point to be examined is that of 180°, which must be done in the usual way, by bringing the points of 0 and 180° to the moveable wires of the opposite micrometers, and then turning the circle half-way round, and bisecting the points

again with the moveable wires; and lastly, taking half the
difference betwixt the distances of the wires in the two posi-
tions of the circle for the error at the point of 180°. Having
now bisected the point of *zero* with the moveable wire of the
micrometer, which is intended to be used in the rest of the
process (for we shall have no further occasion for both), we
must slide the microscope along the arc, till by moving the
wire a little we can bisect the point of 90°, and then the mi-
crometer must be firmly clamped to the arc. The circle must
then be turned till the point of 180° is brought to the micro-
scope, and that of 90° to the micrometer, so that we may be
able to bisect each by a slight motion of their respective wires.
This being done, we must observe, from the positions of the
wires, how much the interval betwixt them has increased or
decreased in the measurement of the new arc; and this in-
crease or decrease must be noted down with a $+$ or $-$ ac-
cordingly. In the same manner we must proceed through
the remaining two arcs of 90°, observing and noting down
the difference betwixt each and the original arc.

The point of *zero* must now be brought again to the mi-
crometer, and bisected by the moveable wire, and the micro-
scope be made to slide back along the arc, till by moving the
wire a little we can bisect the point of 60°, and when this is
done, the microscope must be clamped. We must then mea-
sure the arc of 60° against every succeeding arc of 60° in the
circle, precisely in the same way that we measured the first
arc of 90° against the other three. The arc of 45° is next to
be measured against every succeeding arc of 45°, and this will
complete all that is necessary to be done in the early part of
the morning before the heat of the sun can have affected the

temperature of the instrument. The rest may be performed at our leisure.

You will immediately perceive the object of this kind of measurement. It enables us to determine, with any degree of accuracy that may be required, the proportion which the first and every succeeding arc of the circle, contained betwixt the micrometer and the microscope, bears to the whole circle, and of course the absolute length of the arcs themselves. Let $a$ denote the real length of the first of these, and $\pm a', \pm a'', \pm a'''$, &c. the difference betwixt the first and second, the first and third, &c. respectively; let A represent any other arc whose length is known, and which is a multiple of $a$, as marked upon the instrument, and let this multiple be expressed by $n$. Then will $a + (a + a') + (a + a'') + (a + a''') + $ &c. ...

$(a + a''''^{\cdots \overline{n-1}}) = A$, and $a = \frac{A - a' - a'' - \cdots a'''^{\cdots \overline{n-1}}}{n}$. Hence it

is evident, that if there is no error committed in the measurement of any of these arcs, we shall have the value of $a$, and consequently of $a + a', a + a'', a + a'''$, &c. and of any arc, comprehending any number of these, accurately determined. But if there be an error of $e$ in the measurement of the first, of $e', e'', e'''$, &c. in the measurement of the second, third, &c. respectively, then we shall have the following equation for determining $a$, viz. $a + (a + a' + e + e') + (a + a'' + e$

$+ e'') + $ &c. ... $(a + a''''^{\cdots \overline{n-1}} + e + e'''^{\cdots \overline{n-1}}) = A$, and

consequently $a$ will appear to be equal to $\dfrac{A - a' - a'' - \cdots a'''^{\cdots \overline{n-1}}}{n}$

$\dfrac{-\overline{n-1}e - e' - e'' - \cdots e'''^{\cdots \overline{n-1}}}{n}$, which differs from its true value by

$\dfrac{\overline{n-1}e + e' + e'' + \cdots e'''^{\cdots \overline{n-1}}}{n}$. Hence it follows, that the value of

the $p^{tb}$ arc ($p$ being greater than unity), as deduced by this pro-

cess, will differ from its true value by $\dfrac{\overline{n-1}\cdot e+e'+e''+\dots e'''\cdots\overline{p-1}}{n}$

$+\dfrac{e'''\cdots p+\dots e'''\cdots\overline{n-1}}{n}-e-e'''\cdots\overline{p-1}$ , and that if we add any num-

ber $p$ of these arcs together, in order to determine the value
of the arc which is equal to their sum, we shall have an error
in this value (and the expression holds when $p$ is unity, or

the first arc only is taken) equal to $p\,\dfrac{\overline{n-1}\cdot e+e'+e''+\dots e'''\cdots\overline{p-1}}{n}$

$+\dfrac{e''\cdots p+\dots e''\cdots\overline{n-1}}{n}-\overline{p-1}\cdot e-e'-e''-\dots e'''\cdots\overline{p-1}=\overline{n-p}\cdot$

$\dfrac{\overline{e-e'-e''-\dots e'''\cdots\overline{p-1}}+p\cdot e'''\cdots p+e'''\cdots\overline{p+1}+\dots e''\cdots\overline{n-1}}{n}.$

Now, if we suppose $e$ to be the greatest error to which
we are liable in the measurement of any arc, and each
of the succeeding errors to be equal to it, and likewise
that $e',e'',\dots e'''\cdots\overline{p-1}$ are all negative, then it will appear that
$\dfrac{n-p}{n}\times 2pe$ will be the greatest error that can be committed in
determining the value of any arc, by adding together the
values of the ($p$) smaller arcs of which it is compounded.  For
instance, if the interval betwixt the micrometer and the mi-
croscope comprehends an arc of 60°, as marked upon the in-
strument, and this arc is measured against every succeeding
arc of 60° in the whole circle, we shall have the greatest error
that can be committed in deducing the arc of 120° from the
addition of the two first arcs of 60°, equal to $\dfrac{6-2}{6}\times2\times2e=2.66e.$

After these remarks, we may proceed to consider how the
remaining divisions upon the circle may be examined with the
least probable error, and to ascertain the amount of the great-
est to which the process can in any case be liable.

Let the arc of 30° be now measured against every succeeding arc of 30° in the first, third, fourth, and sixth arcs of 60°, and let the length of each be determined from a separate comparison with the arc of 60°, in which it is comprehended, and not from a general comparison with all the four. The arc of 15° must then be measured against every succeeding arc of 15° in the first, third, fourth, sixth, seventh, ninth, tenth, and twelfth arcs of 30°, and the value of each deduced from a comparison with the arc of 30°, in which it is contained. When this is done, we shall have determined the length of every succeeding arc of 15°, of the first arcs of 30, 45, 60, 75 ($=60+15$), 90, 105 ($=90+15$), 120 ($=60+60$), 135 ($=90+45$), 150 ($=120+30$), 165 ($=150+15$), and 180° in each semi-circle.

We must next measure the arc of 5° against every succeeding arc of 5° in the whole circle, and deduce the values of the first, and of the sum of the first and second, in each succeeding arc of 15° from a comparison with the arc of 15° in which they are contained. We must then proceed to determine the values of the first arc of 3° in each 15°, and of its multiples the arcs of 6, 9, and 12°. We must also put down the value of the last arc of 3° in each arc of 15°, and then deduce the values of the first and last arcs of 1° in each arc of 15°, from a comparison with the arc of 3° in which they are respectively contained.

We shall now have measured in each arc of 15° the first arcs of 1, 3, 5, 6, 9, 10, 12°, and by taking the last arc of one degree, which has likewise been determined, from the arc of 15°, we shall obtain the first arc of 14°. The first 7° of this arc being measured against the second, we ascertain the value of the first 7°; and then, by measuring the first 4° of the re-

maining arc of 8° against the second, we shall get the value
of the first 4°, which added to the arc of 7°, before determined,
will give us the length of the first arc of 11°. The first 2° of
the remaining arc of 4° must then be measured against the
second, and we shall get the value of the first 2°, and by add-
ing this arc to the arc of 11°, we shall obtain the value of the arc
of 13°. By taking away the first arc of 1° from the arc of 15°,
we get the remaining arc of 14°, and then having determined
the length of the first 7° of this arc, by measuring them
against the second, we must add it to the arc of 1°, and we
shall obtain the arc of 8°. The length of the first 4° of this
arc will then be easily known, by measuring them against the
second, as will afterwards that of the first 2° in the arc of 4°
itself, by measuring them against the second in the same arc.

We have still to ascertain the lengths of all the first arcs
of 10, 20, 30, 40, and 50 minutes contained in each degree,
for I shall only consider the case in which the circle is divided
into parts of 10 minutes. Now the length of the first arc of
30′ will be obtained by measuring it against the second, and
the lengths of the first and second arcs of 20′ (whose sum
will give the arc of 40′) by measuring the first against each
of the remaining arcs. The length of the third arc of 20′
must likewise be put down, and then the first arc of 10′ being
measured against the second of the arc of 20′, in which it is
included, and also against the two arcs of 10′ contained in
the last arc of 20′, its own value, and that of the last 10′ in the
degree will be determined from a comparison with the arcs
of 20′, in which they are respectively comprehended. The
length of this last arc of 10′ being taken from that of the
whole degree, will give us the length of the first 50′, and
complete the operation.

In order to ascertain the greatest possible error to which we are liable in the examination, let $\varepsilon$ denote in parts of a second the greatest that can be committed in bisecting any point upon the limb; then, since this error may occur at each end of the arc, it is evident that $e$ in the expression deduced above ($\frac{n-p}{n} \times 2pe$) will become $2\varepsilon$, and the expression itself $\frac{n-p}{n} \times 4p\varepsilon$. Hence the possible error will be $\frac{2-1}{2} 4\varepsilon = 2\varepsilon$ at $180°$;

$\frac{2\varepsilon}{2} + \frac{2-1}{2} \times 4\varepsilon = 3\varepsilon$ at $90°$; $\frac{2\varepsilon}{3} + \frac{3-1}{3} \times 4\varepsilon = 3.33\varepsilon$ at $60°$; $\frac{2}{3} \times 2\varepsilon + \frac{3-2}{3} \times 4 \times 2\varepsilon = 4\varepsilon$ at $120°$. The greatest error must therefore lie betwixt $90$ and $120°$, and nearer to the extremity of the latter than of the former arc. At $105°$ it will be $5.50\varepsilon$; at $111°$ it will be $5.50\varepsilon - \frac{2}{5} \cdot 1.5\varepsilon + \frac{5-2}{5} \times 4 \times 2\varepsilon = 9.70\varepsilon$; and at $111°$ $10'$ it will be $9.70\varepsilon - \frac{1}{6} \cdot 1.04\varepsilon$ (the excess of the error at $111°$ above that at $112°$) $+ 3.33\varepsilon = 12.86\varepsilon$, which will be found to be the greatest error betwixt $105$ and $120°$, and of course the greatest in the first semi-circle. In the other semi-circle, the process being the same, the possible errors must necessarily be the same at the same distances from the first point, reckoning the contrary way upon the circle.

The magnitude of the quantity $\varepsilon$ will of course vary upon circles of the same radius, according to the excellence of the glass employed, and the accuracy of the examiner's eye. It will seldom, however, exceed one second upon a circle, whose radius is one foot; and in general it will not amount to so much. I find that I can read off, to a certainty, within less than three fourths of a second, and hence I conclude, that I could examine the divisions of my circle without being liable to a greater error than 9.63 seconds, and those of a circle of

three feet radius without the risk of a greater error than 3,21 seconds.

To those people who are accustomed to entertain such exalted notions of the accuracy with which astronomical instruments can with a certainty be divided, this error, I dare say, will appear very considerable ; but for my part, I am perfectly satisfied that it bears but a small proportion to the accumulated error which may take place, in spite of the utmost vigilance of the artist, in an instrument divided according to any method which has hitherto been made public. I need not, however, remark upon the very great improbability that the error of examination should ever attain, or approach, to its extreme limit, as this must be sufficiently obvious to any person who is in the least degree conversant with the doctrine of chances ; but it may be proper to observe, that we have it in our power (and in this respect the examiner possesses a most important advantage over the divider of an instrument) to diminish its probable amount, as much as we please, by bringing the moveable wires of the micrometer and microscope several times to bisect their respective points in the measurement of every arc, and taking a mean of the different *readings-off* for the true position of the wire at the real bisection of the point. The wire may be moved in this manner eight or ten times at each point (if such a degree of caution should be thought necessary), and the mean taken in little more than a minute, so that the time of performing the work will not be so much increased, as might perhaps have been apprehended, and when it is completed, we may reasonably presume that the distance of every point from zero ( whilst the temperature of the circle continues uniform) will have been

determined with sufficient exactness for every practical pur-
pose.

Of the time necessary for the examination, a pretty correct
idea may be formed by considering how many measurements
are required, and allowing about a minute and a half for each;
*i. e.* a quarter of a minute for bringing the extreme points of
the arc to the micrometer and the microscope, and a minute
and a quarter for making the several bisections. Now,
in dividing the whole circle into arcs of 15° each, it will ap-
pear that forty-four measurements must be performed; and
to examine every point in each arc of 15°, there will be 161
required, making in all 3908 measurements; and consequently
the time necessary for completing the whole work will be
5862 minutes, or about 98 hours.

The time and labour required for this examination are, no
doubt, very considerable; but it ought to be recollected, that
it will render any great degree of precision, in dividing the in-
strument, totally unnecessary. Whoever indeed employs this
method of examinination will be virtually the divider of his own
instrument, and all that he will ask of the artist, is to make him
a point about the end of every five or ten minutes, whose dis-
tance from zero he will determine for himself, and enter in
his book to be referred to when wanted. We may likewise
observe that by this examination we shall not only be se-
cured against the errors of division, but against those which
arise from bad centering, and from the imperfect figure of
the circle, and which in general are of too great a magnitude
to be neglected.

It will, I dare say, have occurred to you, that whenever we
are desirous that an observation should be particularly exact,

we may guard it against the effects of unequal expansion or contraction in the metal, by means of the apparatus which I have described: for we have only to measure the arc which has been determined by the observation against the whole circle, or against the multiple of it, which approaches nearest to the circle, and from thence to deduce its value in the manner explained above, and we shall either have entirely excluded the error which we apprehended, or have rendered it too small to be of any importance. Suppose, for instance, that the arc determined by the observation was 48°; then by measuring it against the whole circumference increased by an arc of 24°, we shall obtain a result free from any greater error of unequal temperature, than one eighth of the increase or decrease of this arc of 24° beyond a due proportion to that of the circle itself.

This expedient gives us all the advantages of the French circle of repetition, without the inconvenience arising from being obliged to turn the instrument, and move the telescope, so many times in the course of the observation. Nay, I am persuaded, that the result may be made more accurate in this way, than by the French method, because not only can the object be more frequently observed, but the contacts or bisections, it may be presumed, will be more exact when the observer is not disturbed by the hurry attendant upon the use of the repeating circle; and with respect to any error in the instrument, from whatever cause it may arise, it will be as effectually excluded by the process which I recommend, as by moving the telescope round the circle. Besides this method is applicable either to the azimuth or altitude circle, or indeed to any circle which turns upon its own axis, whereas

the French method can never be applied to the azimuth circle, nor to any other circle which does not turn both upon its own axis, and upon one which is perpendicular to it.

After all, however, it is possible that the process which I have been explaining to you may be no new discovery, and that you may be already acquainted with it. If this should be the case, you will be kind enough to inform me. At any rate, indeed, I should esteem myself greatly obliged, if you would favour me with your sentiments upon the subject, as soon as you can do it with perfect convenience to yourself.

I am, Dear Sir,

yours, &c.

WILLIAM LAX.